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THE NONCENTRAL CHI-SQUARED DISTRIBUTION WITH ZERO DEGREES OF FREEDOM AND TESTING FOR UNIFORMITY

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ABSTRACT

The noncentral chi-squared distribution with zero degrees of freedom is defined as a Poisson mixture of mass at zero together with chi-squared distributions that have even degrees of freedom. Their name is justified by the decomposition of the classical noncentral chi-squared distribution as the sum of a central chi-squared component having the full number of degrees of freedom and an independent noncentral chi-squared component having zero degrees of freedom. The basic properties of this one-parameter family of distributions are given, and they are shown to be useful in the computation of approximate critical values of a test for uniformity.

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SIGNIFICANCE AND EXPLANATION

The chi-squared family of statistical tests and probability distributions is the basis for many tests of significance and goodness-of-fit in statistics. This paper reports the discovery of a new distribution from this family: the noncentral chi-squared distribution with zero degrees of freedom. This distribution cannot be defined in the conventional way, which explains why it was unnoticed until now. However, it can be properly defined in another way, and it leads to the previously impossible decomposition of the classical noncentral chi-squared distribution into two parts: a completely central component with all of the degrees of freedom, and a completely noncentral component with no degrees of freedom.

The distribution is also useful in its own right in connection with testing the hypothesis that given observations $X_1,\ldots,X_n$ all between 0 and 1 are independently chosen from the uniform distribution in $(0,1)$. An application is outlined in conjunction with the improvement upon Sir R. A. Fisher's test for periodicity in a time series reported in MRC Technical Summary Report #1843. The noncentral chi-squared distribution with zero degrees of freedom provides much better approximate critical values, necessary for the use of this test, than does the usual Normal or Gaussian distribution approximation.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

THE NONCENTRAL CHI-SQUARED DISTRIBUTION WITH ZERO DEGREES OF FREEDOM AND TESTING FOR UNIFORMITY

Andrew F. Siegel*

1. The Noncentral Chi-Squared Distribution with Zero Degrees of Freedom: Definition and Basic Properties

The central and noncentral chi-squared distributions are fundamental tools in many areas of theoretical and applied statistics (Lancaster (1969)). In this paper, attention is focused on an unexplored group of distributions from this family: those with zero degrees of freedom. Their definition and basic properties are given in this section, and an example of their use in testing for uniformity is given in Section 2. It is expected that many additional applications will be found in the future.

There are several reasons why the case of zero degrees of freedom has been overlooked since Fisher, in 1928, first derived the noncentral chi-squared distribution. First, it does not possess a probability density function because of a discrete mass point at zero. Second, it cannot be defined simply as the sum of independent squared normal deviates with variance one. And third, the central chi-squared distribution with zero degrees of freedom is identically zero, wrongly suggesting that the general case of zero degrees of freedom would be trivial.

The noncentral chi-squared distribution, $\chi_0^2(\lambda)$, with zero degrees of freedom and noncentrality parameter $\lambda \geq 0$ is most directly approached as a compound Poisson mixture of central chi-squared distributions with even degrees of freedom. This extends the standard representation (for example on page 132 of Johnson and Kotz, (1970)) to the case of zero degrees of freedom. We define

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$$Y_\lambda \sim \chi_0^2(\lambda)$$

to be the result of the two-stage process in which we first choose K from a Poisson distribution with mean $\lambda/2$ so that

$$P(K=k) = e^{-\lambda/2}(\lambda/2)^k/k!,\ k = 0,1,2,\ldots$$

and then choose

$$Y_\lambda \sim \chi_{2K}^2 .$$

When K = 0, we adopt the convention that the (central) χ_0^2 distribution is identically zero; this accounts for the discrete component of the $\chi_0^2(\lambda)$ distribution. Thus $\chi_0^2(\lambda)$ is a mixture of the distributions 0, χ_2^2, χ_4^2, χ_6^2,... with weights $\exp(-\lambda/2)$, $\exp(-\lambda/2)(\lambda/2)$, $\exp(-\lambda/2)(\lambda/2)^2/2$, $\exp(-\lambda/2)(\lambda/2)^3/6,\ldots$.

The basic properties of this distribution can be derived directly from the compound Poisson representation. The characteristic function, reproductivity properties, moments, cumulants, asymptotic behavior, cumulative distribution function, and density (to the extent that one exists) will be exhibited in the remainder of this section.

The characteristic function of $Y_\lambda \sim \chi_0^2(\lambda)$ is

$$\phi_\lambda(t) = E\exp(itY_\lambda) = \exp\{it\lambda(1-2it)^{-1}\} \tag{1.1}$$

which is obtained from the Poisson mixture of the characteristic functions $(1-2it)^{-k}$ of the χ_{2k}^2 distributions. This is the same formula obtained by substituting zero for the degrees of freedom in the characteristic function of the classical noncentral χ^2 distribution.

The reproductive properties of this distribution follow immediately from its characteristic function (1.1). Let "*" denote the convolution operator, so that F*G denotes the distribution of X+Y where X and Y are independent random variables chosen from the distributions F and G respectively. Then we have

$$\chi_0^2(\lambda_1)*\chi_0^2(\lambda_2) = \chi_0^2(\lambda_1+\lambda_2) \tag{1.2}$$

$$\chi_0^2(\lambda)*\chi_n^2 = \chi_n^2(\lambda) \tag{1.3}$$

$$\chi_0^2(\lambda_1)*\chi_n^2(\lambda_2) = \chi_n^2(\lambda_1+\lambda_2)\ . \tag{1.4}$$

(1.3) is of particular interest because it allows us to decompose the $\chi_n^2(\lambda)$ distribution into a complete central part with the full n degrees of freedom and a noncentral part without any degrees of freedom. Thus $X\sim\chi_n^2(\lambda)$ can be decomposed as $X=Y+Z$ where $Y\sim\chi_0^2(\lambda)$ and $Z\sim\chi_n^2$ are independent. Going as far back as p. 669 of Fisher (1928), the $\chi_n^2(\lambda)$ is traditionally decomposed as a convolution of $\chi_1^2(\lambda)$ and χ_{n-1}^2, often (as in Theorem 1.1 on page 117 of Lancaster, 1969) by representing it as the sum of n independent squared normal deviates with variance one and using a rotation in n-space to bring the mean vector to the first coordinate axis. This confounding of one degree of freedom with the noncentrality is no longer necessary; a complete separation of noncentrality from all degrees of freedom is now possible.

The cumulants of $Y_\lambda \sim \chi_0^2(\lambda)$ are seen from (1.1) to be $\kappa_m = \lambda 2^{m-1}m!$. The moments can be found directly from a Poisson mixture of the moments of the component central χ^2 distributions. The moments are

$$EY_\lambda^m = 2^m m! \sum_{k=1}^{m} \binom{m-1}{k-1} (\lambda/2)^k/k! \ . \tag{1.5}$$

Moments and cumulants of low order are

m	moment	central moment	cumulant
1	λ	0	0
2	$\lambda^2+4\lambda$	4λ	4λ
3	$\lambda^3+12\lambda^2+24\lambda$	24λ	24λ
4	$\lambda^4+24\lambda^3+144\lambda^2+192\lambda$	$192\lambda+48\lambda^2$	192λ

(1.6)

Asymptotic normality of $Y_\lambda \sim \chi_0^2(\lambda)$ holds when λ is large. More precisely,

$$\frac{Y_\lambda-\lambda}{2\sqrt{\lambda}} \xrightarrow{D} N(0,1) \qquad \text{as } \lambda \to \infty \tag{1.7}$$

which is quickly proven using the characteristic function (1.1).

Asymptotically when λ is small, the positive component of $Y_\lambda \sim \chi_0^2(\lambda)$ tends to χ_2^2. Noting that most of the mass is at zero in this case, we decompose Y_λ into the mixture

$$Y_\lambda = \begin{cases} 0 & \text{pr} \quad \exp(-\lambda/2) \\ Z_\lambda & \text{pr} \quad 1-\exp(-\lambda/2) \end{cases} \tag{1.8}$$

so that Z_λ is the conditional random variable $Y_\lambda | \{Y_\lambda > 0\}$, which is positive and continuous. In fact, Z_λ is the mixture of $\chi_2^2, \chi_4^2, \chi_6^2, \ldots$ with mixing probabilities $\mathrm{pr}(K=k|K>0)$ where $K \sim P_0(\lambda/2)$. Then using the decomposition (1.8) and the characteristic function (1.1) one can show that

$$Z_\lambda \xrightarrow{D} \chi_2^2 \qquad \text{as} \qquad \lambda \to 0 \ . \tag{1.9}$$

The cumulative distribution function of $Y_\lambda \sim \chi_0^2(\lambda)$ is

$$F_\lambda(t) = \mathrm{pr}(Y_\lambda \leq t) = 1 - e^{-(\lambda+t)/2} \sum_{k=1}^{\infty} \frac{(\lambda/2)^k}{k!} \sum_{\ell=0}^{k-1} \frac{(t/2)^\ell}{\ell!} \qquad (1.10)$$

when $t \geq 0$ and is zero otherwise. These series converge quickly; hence this formula is convenient for computing. Figure 1 shows the cumulative distribution function $F_\lambda(t)$ of $\chi_0^2(\lambda)$ for various values of λ. Clearly apparent are the discontinuities at $t=0$ (due to the mass $\exp(-\lambda/2)$ at zero), asymptotic normality when λ is large, and asymptotic exponentiality (χ_2^2) of the positive component when λ is small.

The density of the $\chi^2_0(\lambda)$ distribution, properly speaking, does not exist due to the mass at zero. However, the positive part of this distribution does have a "density" $f_\lambda(t)$ in the sense that if $Y_\lambda \sim \chi^2_0(\lambda)$ and $0 \leq a < b$, then

$$P(a < Y_\lambda < b) = \int_a^b f_\lambda(t)\,dt.$$

Mixing the densities of the non-degenerate component central χ^2, we find that

$$f_\lambda(t) = \frac{1}{t}\, e^{-(\lambda+t)/2} \sum_{k=1}^{\infty} \frac{(\lambda t/4)^k}{k!(k-1)!} \qquad (1.11)$$

which can also be expressed as

$$f_\lambda(t) = \frac{1}{2} \surd(\lambda t) e^{-(\lambda+t)/2} I_1\{\surd(\lambda t)\}$$

where I_1 denotes the first modified Bessel function. This is not a true density because its total mass is only

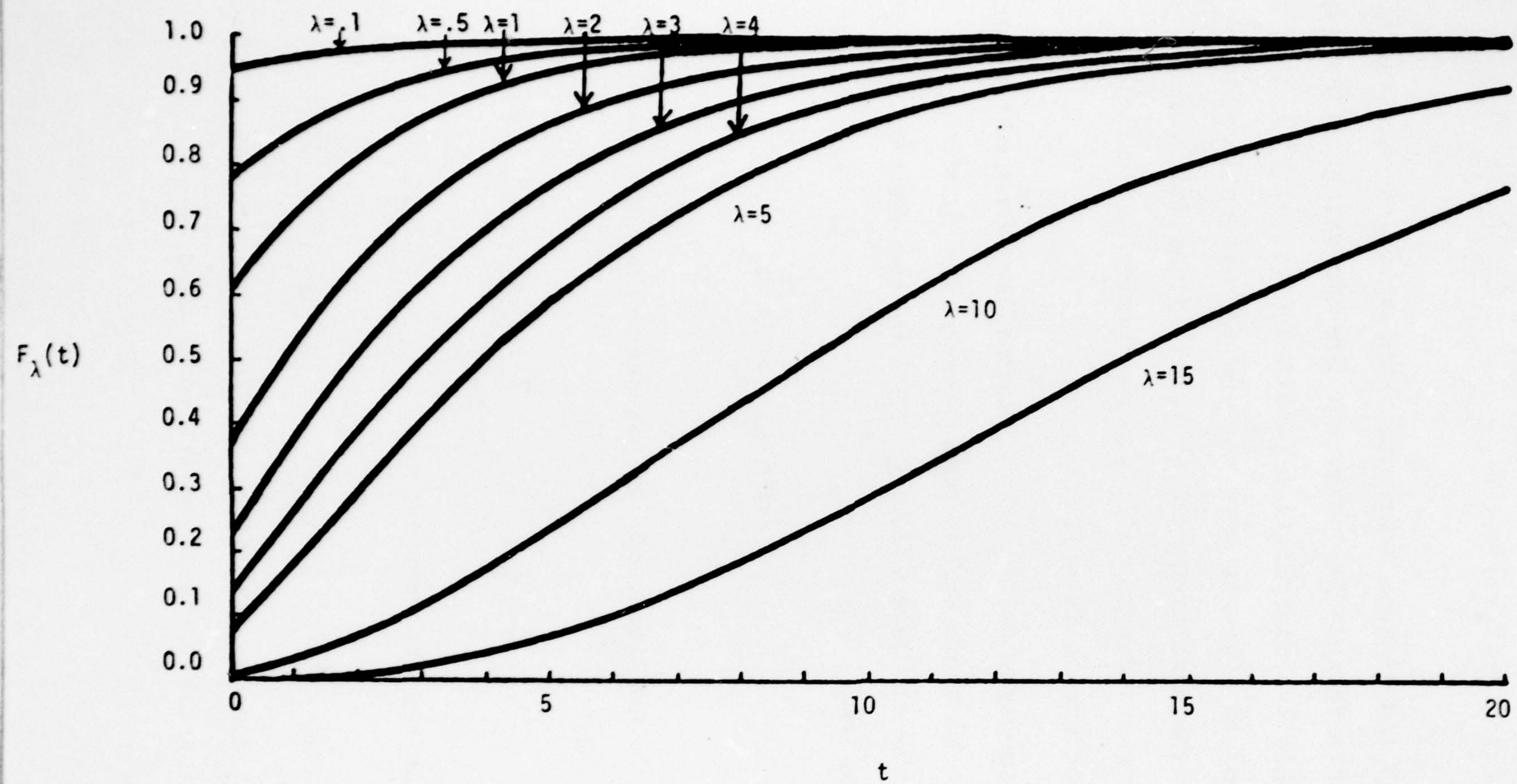

Figure 1. Cumulative Distribution Function $F_\lambda(t)$ of the $\chi^2{}_0(\lambda)$ Distribution Plotted for Some Values of the Noncentrality Parameter, λ.

$$P(0<Y_\lambda<\infty) = \int_0^\infty f_\lambda(t)dt = 1 - \exp(-\lambda/2)<1 \quad .$$

If we normalize f_λ and define

$$g_\lambda(t) = f_\lambda(t)/(1-\exp(-\lambda/2)) \tag{1.12}$$

then $g_\lambda(t)$ is a true density. It is the density of $Z_\lambda = Y_\lambda|\{Y_\lambda>0\}$, the positive continuous random variable defined in (1.8).

Graphs of these "densities" $f_\lambda(t)$ are shown in Figures 2 and 3. Figure 2 shows the case $\lambda\leq2$, and we see clearly that they are not true densities because the areas under the curves are not equal. This is because as λ increases, the mass $\exp(-\lambda/2)$ at zero decreases and is moved to the right (to the positive continuous part) increasing the area $(1-\exp(-\lambda/2))$ under these curves. Again we note the exponential (χ^2_2) form of these curves when λ is small, as was shown in (1.9).

The ordinate intercept $f_\lambda(0)$ takes its maximum value of $1/(2e)$ at $\lambda=2$, and Figure 3 shows some "densities" $f_\lambda(t)$ when $\lambda\geq2$ and this intercept is decreasing in λ. When $\lambda=10$, only a mass of .00674 remains at zero, and we begin to see the trend towards asymptotic normality for large λ predicted by (1.7).

2. Testing for Uniformity

The purpose of this section is to show how the noncentral chi-squared distribution with zero degrees of freedom can be used in the approximation of critical values for a test of uniformity.

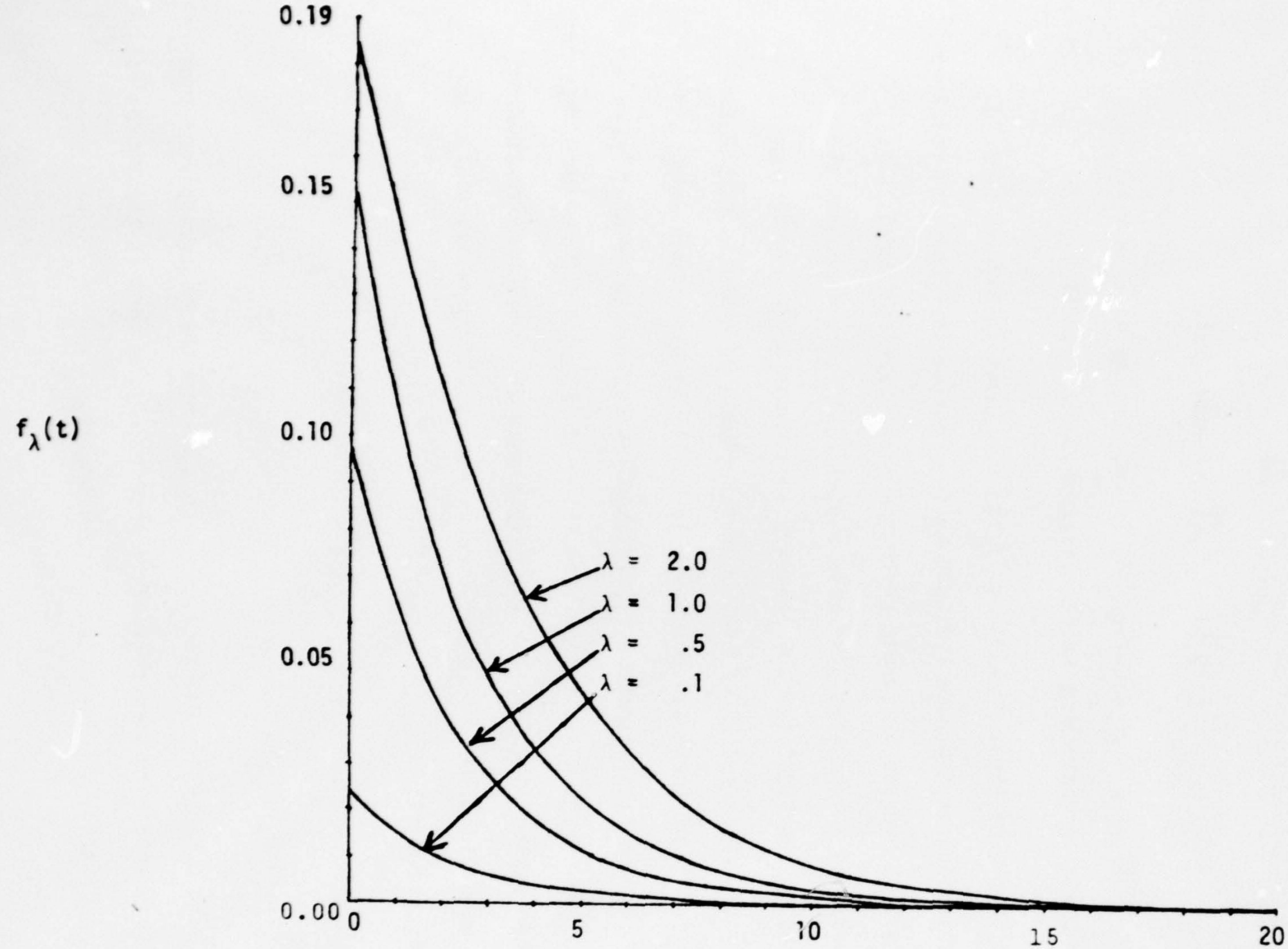

Figure 2. Improper Density $f_\lambda(t)$ of the $\chi^2{}_0(\lambda)$ Distribution Plotted for Some Values of the Noncentrality Parameter λ, for $\lambda \leq 2$.

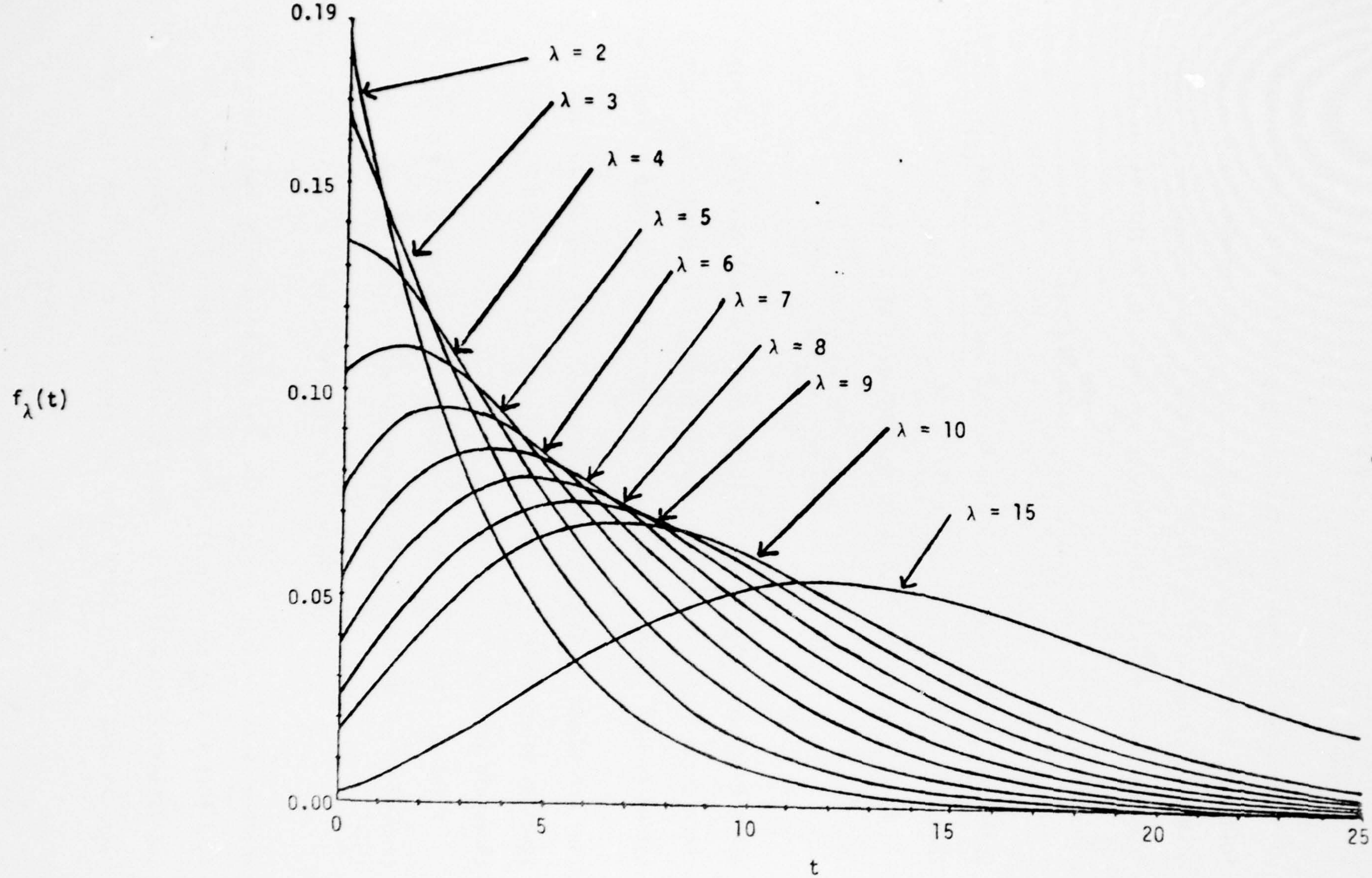

Figure 3. The Improper Density $f_\lambda(t)$ when $\lambda \geq 2$.

Many problems involving testing a hypothesis can be reduced to the following situation: given data $X_1,\ldots,X_{n-1}$ all between 0 and 1, we wish to test the null hypothesis that the X_i were independently chosen from the uniform distribution on the interval (0,1). That is, test

$$H_0: X_1,\ldots,X_{n-1} \overset{\text{iid}}{\sim} \mathcal{U}(0,1).$$

Order the data and adjoin the endpoints to obtain $0 = X_{(0)}<X_{(1)}<\ldots<X_{(n-1)}<X_{(n)}=1$. Then define the spacings $Y_j = X_{(j)}-X_{(j-1)}$, $j = 1,\ldots,n$.

We will consider tests based on statistics of the form

$$T(n,a) = \sum_{j=1}^{n} (Y_j-a)_+ \tag{2.1}$$

where $0<a<1$ and $(t)_+ = \max(t,0)$ is the positive-part function. I have shown these statistics to be useful in testing for periodicity in a time series (Siegel 1979b). They are sensitive to the existence of large spacings.

They are adaptive and continuous, for they select only the largest spacings (those with $Y_j>a$) and sum the excess of each such Y_j above the threshold value a.

Fisher's (1929) test for periodicity can be obtained as a special case of (2.1) when $a = a_n^*$ is chosen so that

$$P(T(n,a_n^*)>0) = P(\max_j Y_j>a_n^*) = \alpha \tag{2.2}$$

where α is the desired level of the test and a_n^* depends implicitely on α. Note that if $a>a_n^*$ then $T(n,a)$ has mass greater than $1-\alpha$ at zero and randomization will be necessary to insure level α. Thus the nonrandomized level α tests based on statistics of the form (2.1) come from a member of the one-parameter family of statistics $T(n,\zeta a_n^*)$ where $0<\zeta\leq 1$. $\zeta = 1$ yields Fisher's test, while $\zeta = 0$ yields the useless statistic $T(n,0) \equiv 1$. A power study in Siegel (1979b) showed that the choice $\zeta = .6$ yielded a good overall test with significant power gains over Fisher's test against certain alternatives.

The null distribution of this statistic was found to be

$$P(T(n,\zeta a_n^*)>t) = \sum_{\ell=1}^{n} \sum_{k=0}^{\ell-1} (-1)^{k+\ell+1} \binom{n}{\ell}\binom{\ell-1}{k}\binom{n-1}{k} t^k (1-\zeta\ell a_n^*-t)_+^{n-k-1} \tag{2.3}$$

and critical values for n up to 50 were tabled. For large n, the terms with alternating signs can be quite large, leading to a serious problem with round-off error during computation. This is why we seek the asymptotic distribution of this statistic.

There are two candidates for the asymptotic distribution of $T(n,\zeta a_n^*)$: the normal distribution and the $\chi^2_0(\lambda)$ distribution. This follows from Theorems 3.2 and 4.1 of Siegel (1979a) because the distribution of $V(n,a)$ of that paper is identical to the distribution of $T(n,a)$ here. Theorem 4.1 showed that $T(n,\zeta a_n^*)$ actually is asymptotically normal for fixed ζ as $n\to\infty$. However, for even moderately large n, $T(n,\zeta a_n^*)$ can still place significant mass at zero, and the normal approximation may not be very good. Theorem 3.2 allowed ζ to depend on n so that the mass at zero was preserved in the limit, and the $\chi^2_0(\lambda)$ distribution was obtained.

Each of these distributions (normal and $\chi^2_0(\lambda)$) yields an approximate critical value for $T(n,\zeta a_n^*)$, obtained by matching up the first two moments. The first two moments of $T(n,\zeta a_n^*)$ are the same as those calculated for $D(n,\zeta a_n^*)$ in Siegel (1978). Thus

$$ET(n,\zeta a_n^*) = (1-\zeta a_n^*)^n \tag{2.4}$$

and

$$E[T(n,\zeta a_n^*)]^2 = \frac{1}{n+1}[2(1-\zeta a_n^*)^{n+1} + (n-1)(1-2\zeta a_n^*)_+^{n+1}] \ . \tag{2.5}$$

The normal approximation is based on the critical values of a normal distribution with these first two moments. The $\chi^2{}_0(\lambda)$ approximation will be based on the critical values of $c\chi^2{}_0(\lambda)$ where the scale factor c and the noncentrality parameter λ are chosen so that $c\chi^2{}_0(\lambda)$ has (2.4) and (2.5) as its first two moments. Using (1.6) the solution is

$$c = \frac{2(1-\zeta a_n^*)^{n+1}+(n-1)(1-2\zeta a_n^*)_+^{n+1}-(n+1)(1-\zeta a_n^*)^{2n}}{4(n+1)(1-\zeta a_n^*)^n} \tag{2.6}$$

and

$$\lambda = (1-\zeta a_n^*)^n/c \ . \tag{2.7}$$

Critical values for $T(n,\zeta a_n^*)$ with the preferred choice $\zeta = .6$ were calculated exactly using (2.3) and approximately using the normal and the $\chi^2{}_0(\lambda)$ approximations. The results are listed in Table 1 for levels $\alpha = .05$ and .01, and for n = 10 through 50. Comparing these, we see that the $\chi^2{}_0(\lambda)$ approximation is clearly superior to the normal approximation. Note that the critical values from the $\chi^2{}_0(\lambda)$ approximation are very close to the actual critical values, even for relatively small values of n.

The two columns on the right-hand side of Table 1 show that the differences between the two approximations are significant.

When n = 50 the normal approximation with nominal level $\alpha = .05$ has actual level $\alpha = .0774$, and the normal approximation with nominal level $\alpha = .01$ has actual level $\alpha = .0439$. The actual levels obtained using the $\chi^2_0(\lambda)$ approximation are much closer to the nominal levels: they are $\alpha = .0509$ and $\alpha = .00985$ respectively.

The reason why the normal approximation fails here is because $n = 50$ is not yet large enough for those asymptotics to be appropriate. The amount of mass that $T(n,.6a_n^*)$ places at zero will diminish to zero in the limit, but at level $\alpha = .01$ with $n = 50$, there is still a mass of .674 at zero! The $\chi^2_0(\lambda)$ distribution is not affected by this problem because it is, like $T(n,\zeta a_n^*)$ itself, a mixture of mass at zero with positive continuous variation.

The clear recommendation is thus to use $c\chi^2_0(\lambda)$, where c and λ are found from (2.6) and (2.7), as an approximation to the null distribution of $T(n,\zeta a_n^*)$. This will still be able to handle the ultimate asymptotic normality of $T(n,\zeta a_n^*)$ because, by (1.7), $\chi^2_0(\lambda)$ is also asymptotically normal as $\lambda \to \infty$.

Table 1. A comparison of the exact critical values of $T(n,.6a_n^*)$ with the approximations calculated using the normal distribution and the $\chi^2_0(\lambda)$ distribution

			Critical Values			Actual Level	
	n	$.6a_n^*$	Normal	$\chi^2_0(\lambda)$	Exact	Normal	$\chi^2_0(\lambda)$
$\alpha = .05$	10	.267	.151	.178	.181	.0816	.0529
	20	.162	.0971	.114	.116	.0795	.0519
	30	.119	.0742	.0872	.0880	.0785	.0514
	40	.0944	.0611	.0715	.0721	.0778	.0511
	50	.0788	.0524	.0612	.0616	.0774	.0509
$\alpha = .01$	10	.322	.128	.217	.214	.0467	.00951
	20	.198	.0782	.134	.134	.0450	.00979
	30	.145	.0583	.0998	.0993	.0444	.00983
	40	.115	.0472	.0802	.0799	.0441	.00984
	50	.0957	.0401	.0676	.0673	.0439	.00985

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